No.

Date

問 1

1) $\int_{-\frac{a}{2}}^{\frac{a}{2}}\int_{-\frac{a}{2}}^{\frac{a}{2}} \frac{m}{a^2}(x^2+y^2)dxdy$

$=\frac{m}{a^2}\int_{-\frac{a}{2}}^{\frac{a}{2}}\left[\frac{1}{3}x^3+y^2x\right]_{-\frac{a}{2}}^{\frac{a}{2}}dy$

$=\frac{2m}{a^2}\int_{-\frac{a}{2}}^{\frac{a}{2}}\left(\frac{a^3}{24}+\frac{a}{2}y^2\right)dy$

$=\frac{4m}{a^2}\left[\frac{a^3}{24}y+\frac{a}{6}y^3\right]_0^{\frac{a}{2}}$

$=\frac{1}{6}ma^2$

2) 平行軸の定理より

$\frac{1}{6}ma^2+m\left(\left(\frac{a}{2}\right)^2+\left(\frac{a}{2}\right)^2\right)$

$=\frac{2}{3}ma^2$

3) $\frac{2}{3}ma^2\ \alpha=-\left\{\left(\frac{a}{2}\right)^2+\left(\frac{a}{2}\right)^2\right\}m\,g\,\sin\theta$

$\alpha=-\frac{3}{4}g\sin\theta\simeq-\frac{3}{4}g\theta$

$\omega=\frac{\sqrt{3g}}{2}$ より $T=\frac{2\pi}{\omega}=\frac{4\pi}{\sqrt{3g}}$

問 2

1) $m\frac{v^2}{R}=2mg\quad\therefore v=\sqrt{2gR}$

2) 遠心力が大きいため、より広い半径で円運動するようになる。

3) $x(t)=\{R+r(t)\}\cos\theta(t)$

$y(t)=\{R+r(t)\}\sin\theta(t)$

$z(t)=r(t)$

$x'(t)=r'(t)\cos\theta(t)-\{R+r(t)\}\sin\theta(t)\cdot\theta'(t)$

$y'(t)=r'(t)\sin\theta(t)+\{R+r(t)\}\cos\theta(t)\cdot\theta'(t)$

$z'(t)=r'(t)$

$T=\frac{1}{2}m\left((x'(t))^2+(y'(t))^2\right)+\frac{1}{2}\cdot 2m(z'(t))^2$

$=\frac{1}{2}m\left\{(r'(t))^2+\left(R+r(t)\right)\theta'(t)\right)^2\right\}\ \frac{1}{2}m\left(2(r'(t))^2\right)$

$=\frac{1}{2}m\left\{3(r'(t))^2+(R+r(t))^2\theta'(t)^2\right\}$

$V=2mgr(t)$

$\frac{d}{dt}\left(\frac{\partial L}{\partial r'(t)}\right)=\frac{d}{dt}\left\{\frac{1}{2}m\cdot 3\cdot 2r'(t)\right\}$

$=3r''(t)$

$\frac{\partial L}{\partial r(t)}=m(R+r(t))(\theta'(t))^2-2mg$

$\frac{d}{dt}\left(\frac{\partial L}{\partial r'(t)}\right)-\frac{\partial L}{\partial r(t)}=3r''(t)-m(R+r(t))(\theta'(t))^2+2mg=0$

$\frac{d}{dt}\left(\frac{\partial L}{\partial \theta'(t)}\right)=\frac{d}{dt}\left\{m(R+r(t))^2\theta'(t)\right\}$

$=2m(R+r(t))r'(t)\theta'(t)+m(R+r(t))^2\theta''(t)$

$\frac{\partial L}{\partial \theta(t)}=0$

$\frac{d}{dt}\left(\frac{\partial L}{\partial \theta'(t)}\right)-\frac{\partial L}{\partial \theta(t)}=\frac{d}{dt}\left(m(R+r(t))^2\theta'(t)\right)=0$

$(=2m(R+r(t))r'(t)\theta'(t)+m(R+r(t))^2\theta''(t))$

$h=(R+r(t))^2\theta'(t)$ とすると

$3r''(t)-m(R+r(t))\left\{\frac{h}{(R+r(t))^2}\right\}^2+2mg=0$

$r''(t)-\frac{mh^2}{3}\cdot\frac{1}{(R+r(t))^3}-\left(-\frac{2}{3}mg\right)=0$

$r(t)$ に関して積分すると

$\frac{1}{2}(r'(t))^2+\frac{mh^2}{6}\cdot\frac{1}{(R+r(t))^2}-\left(-\frac{2}{3}mgr(t)\right)=E$

$V^*=\frac{mh^2}{6}\cdot\frac{1}{(R+r(t))^2}-\left(-\frac{2}{3}mgr(t)\right)$