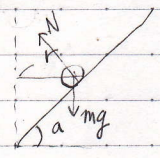


問題4

1) 力のつりあいの式より

$$m \frac{v^2}{r} = N \sin a$$

$$N \cos a = mg$$



よって質点の速度 $v = \sqrt{gr \tan a}$

2) 図より

$$x(t) = r(t) \cos \theta(t)$$

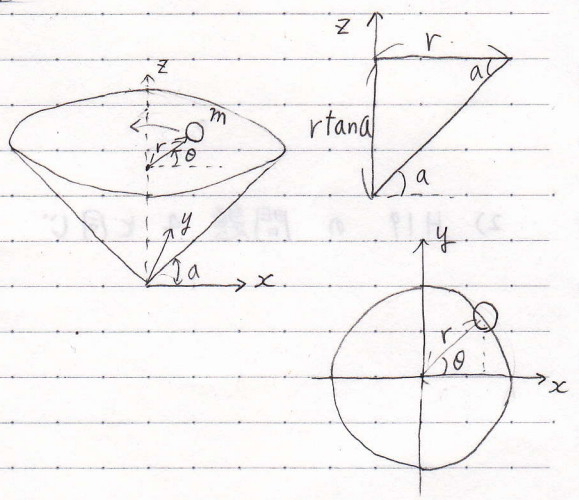
$$y(t) = r(t) \sin \theta(t)$$

$$z(t) = r(t) \tan a$$

$$T = \frac{1}{2} m (\dot{x}(t)^2 + \dot{y}(t)^2 + \dot{z}(t)^2)$$

$$= \frac{1}{2} m \left(\frac{1}{\cos^2 a} \dot{r}(t)^2 + (r(t) \dot{\theta}(t))^2 \right)$$

$$V = mgr(t) \tan a$$



$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{r}(t)} \right) - \frac{\partial L}{\partial r(t)} = \frac{m}{\cos^2 a} \ddot{r}(t) - m r(t) \dot{\theta}(t)^2 - mg \tan a = 0 \quad \text{--- ①}$$

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{\theta}(t)} \right) - \frac{\partial L}{\partial \theta(t)} = m r(t)^2 \dot{\theta}(t) = 0 \quad (\text{角運動量保存則})$$

単位質量あたりの角運動量を $h = r(t)^2 \dot{\theta}(t)$ とおき、①式の $\dot{\theta}(t)$ を消去すると

$$m r \ddot{r}(t) = \frac{m h^2 \cos^2 a}{r(t)^3} + mg \sin a \cos a$$