

問題 4

$$1) \quad x = r \cos \theta, \quad y = r \sin \theta$$

$$\frac{dx}{dt} = \frac{dr}{dt} \cos \theta - r \frac{d\theta}{dt} \sin \theta$$

$$= -v \cos \theta - r w \sin \theta$$

$$\frac{dy}{dt} = \frac{dr}{dt} \sin \theta + r \frac{d\theta}{dt} \cos \theta$$

$$= v \sin \theta + r w \cos \theta$$

$$\frac{d^2x}{dt^2} = -\frac{dv}{dt} \cos \theta + v \frac{d\theta}{dt} \sin \theta - \frac{dr}{dt} w \sin \theta - r \frac{dw}{dt} \sin \theta - r w \frac{d\theta}{dt} \cos \theta$$

$$= v w \sin \theta + v w \sin \theta - r w^2 \cos \theta = \underline{2v w \sin \theta - r w^2 \cos \theta}$$

$$\frac{d^2y}{dt^2} = \frac{dv}{dt} \sin \theta - v \frac{d\theta}{dt} \cos \theta + \frac{dr}{dt} w \cos \theta + r \frac{dw}{dt} \cos \theta - r w \frac{d\theta}{dt} \sin \theta$$

$$= -v w \cos \theta - v w \cos \theta - r w^2 \sin \theta = \underline{-2v w \cos \theta - r w^2 \sin \theta}$$

$$\begin{pmatrix} \frac{d^2x}{dt^2} \\ \frac{d^2y}{dt^2} \end{pmatrix} = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} \frac{d^2x}{dt^2} \\ \frac{d^2y}{dt^2} \end{pmatrix}$$

$$= \begin{pmatrix} -r w^2 \\ -2v w \end{pmatrix}$$

2) x, y 軸方向の運動方程式より

$$m \frac{d^2x}{dt^2} = F_x, \quad m \frac{d^2y}{dt^2} = F_y \quad \therefore F_x = -m r w^2, \quad F_y = -2m v w$$

よって、大きさ $m r w^2$, 円盤の中心方向

$2m v w$, 円盤の回る向きと反対の方向